

# On the Dispersion Measure of High-Redshift Synchrotron Sources

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## ABSTRACT

A method is proposed for deriving the dispersion measure (related to the line-of-sight integrated free electron density) to high-redshift, powerful synchrotron radio sources by analysis of the temporal/spectral properties of the recorded signals. This would allow direct measurements of the distributed density of ionized baryons along the line-of-sights to the sources.

## Key words:

cosmology: miscellaneous — cosmology: observations

## 1 INTRODUCTION

Highly dispersed radio pulses of likely extragalactic origin have been detected at GHz frequencies - one with a dispersion measure  $DM = \int dl n_e = 375 \text{ pc(cm)}^{-3}$  (Lorimer et al. 2007) and another with  $DM = 746 \text{ pc(cm)}^{-3}$  (Keane et al. 2012). Here, the integral is from the earth to the source and  $n_e$  is the free electron density. Unfortunately the pulses do not appear to repeat and the distances to their sources are unknown. Here, we propose that the dispersion measures to high-redshift incoherent synchrotron radio sources can be extracted by analysis of the amplitude cross-correlation of the radio waves received at two nearby frequencies. This method utilizes the broadband nature of the synchrotron radiation.

Section 2 discusses the theory first without intervening plasma and then including it. Also, the influence of interstellar/intergalactic scintillations are considered. Section 3 gives our conclusions.

## 2 THEORY

The measured electric field on the ground from a distant optically-thin incoherent synchrotron source can be approximated as

$$E(t) = \sum_j \mathcal{E}_j \delta(t - t_j), \quad (1)$$

where the different terms represent the contributions from the individual electron or positron in the source,  $\mathcal{E}_j$  is the complex field amplitude, and  $\delta$  is the delta function. The synchrotron radiation from a highly relativistic electron with

Lorentz factor  $\gamma \gg 1$  is directed in a narrow beam of angular width  $\sim 1/\gamma$  in the direction of the electron's instantaneous velocity (e.g., Pacholczyk 1970). This narrow beam sweeps across the observer in a time interval  $dt \sim (\gamma^3 \omega_c)^{-1}$  which is much less than the gyration period of the electron,  $2\pi/\omega_c$ . Here,  $\omega_c = eH/(m_e \gamma c)$  is the relativistic cyclotron frequency with  $e$  and  $m_e$  the charge and rest mass of the electron and  $H$  the magnetic field strength in Gauss. Thus the radiation at a point on the ground is in narrow pulses each of width  $\sim dt$  which is the basis for using delta functions in equation (1). Consequently the frequency spectrum of the radiation is very broad with a maximum at  $\omega_{\text{syn}} \sim (dt)^{-1} = \gamma^3 \omega_c$ . Because the synchrotron radiation is dominantly linearly polarized, we consider  $E$  to be one of the linear polarization components. For the moment we neglect the influence of intervening plasma. The representation of  $E(t)$  by equation (1), where  $\mathcal{E}_j$  has a broad frequency spectrum, is specific to incoherent synchrotron radiation. It does not apply for example to thermal black-body radiation or inverse Compton radiation.

We consider that the signal (1) is first passed through a bandpass filter which passes only frequencies in the interval  $f \pm B/2$ , where  $B \ll f$ . The resulting signal is then mixed with a precise local oscillator reference signal  $\exp(2\pi i f t)$  to give

$$\bar{E}(t) = \sum_j \mathcal{E}_j \Delta(t - t_j), \quad (2)$$

where  $\Delta(\tau)$  is the function which results from passing the delta function through the bandwidth  $B$ . For example, for a flat response within  $B$ ,  $\Delta(\tau) = \sin(\pi B \tau)/(\pi \tau)$  which has

a temporal width  $\sim B^{-1}$ . The expansion (2) is similar that of Appendix A of Cordes et al. (2004).

The average power is proportional to an average of  $|\overline{E}(t)|^2$ . We can assume  $\langle \mathcal{E}_j \mathcal{E}_k^* \rangle = \langle |\mathcal{E}|^2 \rangle \delta_{jk}$  because the different electrons are uncorrelated. The individual pulses at times  $t_j$  are closely spaced with a number per unit time  $\mathcal{N}$  so that we can integrate over  $t_j$  to obtain

$$\langle |\overline{E}|^2 \rangle = \mathcal{N} B \langle |\mathcal{E}_j|^2 \rangle. \quad (3)$$

We have used  $\int_{-\infty}^{\infty} dt [\Delta(t)]^2 = B$  which holds for  $\Delta(t) = \sin(\pi B t)/(\pi t)$ .

Similarly, the autocorrelation functions is found to be

$$\frac{\langle \overline{E}(t) \overline{E}^*(t + \delta t) \rangle}{\langle |\overline{E}|^2 \rangle} = \frac{1}{B} \int_{-\infty}^{\infty} dt \Delta(t) \Delta(t + \delta t) \equiv \rho_E(\delta t), \quad (4)$$

where the asterisk indicates the complex conjugate. Note that  $\rho_E(0) = 1$  and that its width or correlation time is  $\sim B^{-1}$ . For the case  $\Delta(t) = \sin(\pi B t)/(\pi t)$  one finds that  $\rho_E(t) = \Delta(t)/B$ .

Consider the cross-correlation between the fields  $\overline{E}_1$  and  $\overline{E}_2$  at two separated frequencies  $f_1$  and  $f_2$ . The signal  $\overline{E}_2(t)$  is obtained in a way similar to that for  $\overline{E}_1(t)$ , but it is mixed with a second local oscillator signal  $\exp(2\pi i f_2 t)$ , where the frequency  $f_2$  is accurately locked to  $f_1$ . For simplicity we assume both bandwidths are equal to  $B$ . If the two frequencies are close together,  $(f_2 - f_1)^2 \ll f_1 f_2$ , the amplitudes  $\mathcal{E}_j$  do not change significantly between the two frequencies because of the broadband nature of the synchrotron radiation. Thus

$$\frac{\langle \overline{E}_1(t) \overline{E}_2^*(t + \delta t) \rangle}{(\langle |\overline{E}_1|^2 \rangle \langle |\overline{E}_2|^2 \rangle)^{1/2}} \approx \rho_E(\delta t), \quad (5)$$

with  $\rho_E$  given in equation (4).

For the cross-correlation obtained by time-averaging over time intervals  $T$ , the variation in the difference between the two local oscillator frequencies,  $d(f_2 - f_1)$ , must satisfy  $d(f_2 - f_1) < (2\pi T)^{-1}$ ,

$$(6)$$

in order for the correlation not to be smeared out. Thus the local oscillators will need to have very high stability possible with hydrogen maser sources.

## 2.1 Intervening Plasma: Dispersion

Propagation of the radio waves from a distant high redshift source such as a quasar will modify the cross-correlation because of the frequency dependent delay time delay through the ionized intergalactic plasma. Ioka (2003) and Inoue (2004) discuss this intergalactic delay taking into account the redshifting of the frequency and the time dilation during propagation through a homogeneously distributed fully ionized plasma assuming a flat cosmology and express the delay as

$$\tau(f) = \frac{e^2}{2\pi m_e c^2 f^2} \text{DM}, \quad (7)$$

where DM is the dispersion measure,

$$\text{DM} = \int_0^z dz' c \left| \frac{dt}{dz'} \right| \frac{n_e(z')}{1+z'}, \quad (8)$$

where  $|dt/dz| = [(1+z)H(z)]^{-1}$  and  $H(z) = [\Omega_m(1+z)^3 + \Omega_\Lambda]^{1/2}$ . Assuming the plasma is without loss or gain over

time, the electron density varies as  $n_e(z) = n_{e0}(1+z)^3$  with  $n_{e0} = \text{const}$ . Numerically,

$$\tau(f) \approx 4.14 \times 10^{-5} \frac{\text{DM}}{f_{10}^2} \text{ s}, \quad (9)$$

with DM in units of  $\text{pc}(\text{cm})^{-3}$  and  $f_{10}$  is the frequency in units of  $10^{10}$  Hz. For redshifts  $z$  small compared with unity  $\text{DM} = \int d\ell n_e$  where  $d\ell$  is the path element to the source.

Ioka (2003) and Inoue (2004) have evaluated the redshift integral (7) for a homogenous intergalactic medium and estimate that  $\text{DM} \approx 1200z \text{ pc}(\text{cm})^{-3}$  for  $z \lesssim 6$ . Additionally, there may contributions to DM due to propagation through the free electrons of our Galaxy and/or the source galaxy. For our galaxy estimates of the DM in directions perpendicular/parallel to the disc plane range from 30 to  $1000 \text{ pc}(\text{cm})^{-3}$  (Taylor et al. 1993).

In order for the pulse-like nature of the synchrotron radiation not be smeared out by the variation of  $\tau(f)$  across the receiver bandwidth  $B$ , we must have  $B|d\tau/df| < B^{-1}$ . This is the same as the requirement

$$B < 1.1 \times 10^7 \frac{f_{10}^{3/2}}{\text{DM}^{1/2}} \text{ Hz} \approx 3.16 \times 10^5 \frac{f_{10}^{3/2}}{z^{1/2}} \text{ Hz}, \quad (10)$$

where the last expression uses  $\text{DM} \approx 1200z$ . We assume that  $B$  satisfies this condition.

Consider again the cross-correlation of the fields at two nearby frequencies,  $f_1$  and  $f_2 = f_1 + \delta f$ , again with equal bandwidths  $B$ . We have in place of equation (5),

$$\frac{\langle \overline{E}_f(t - \tau_f) \overline{E}_{f+\delta f}^*(t + \delta t - \tau_{f+\delta f}) \rangle}{\langle |\overline{E}_f|^2 \rangle} = \rho_E(\delta t - \tau'_f \delta f). \quad (11)$$

Here,  $\tau_{f,f+\delta f} = \tau(f, f + \delta f)$  and  $\tau'_f = d\tau(f)/df$ . The maximum of  $\rho_E$  occurs at  $\delta t = \tau'_f \delta f$ . Thus measurements of the values  $(\delta t_m, \delta f_m)$  of the maximum allows one to derive  $\tau'_f = \delta t_m / \delta f_m$  which is directly related to the dispersion measure.

From equation (9) we have

$$\tau'_f = -8.28 \times 10^{-15} \frac{\text{DM}}{f_{10}^3} \approx -10^{-11} \frac{z}{f_{10}^3} \text{ s}^2. \quad (12)$$

In order for the shift of the cross-correlation to be detectable with dispersion included we need  $|\tau'_f| \delta f > B^{-1}$ . In view of inequality (10) this implies that we need

$$\delta f > 1.1 \times 10^7 \frac{f_{10}^{3/2}}{\text{DM}^{1/2}} \approx 3.16 \times 10^5 \frac{f_{10}^{3/2}}{z^{1/2}} \text{ Hz}, \quad (13)$$

which is the reverse of inequality (10).

The small bandwidths indicated by inequality (10) may lead to poor signal to noise ratios in measurement of the cross-correlation functions. This may be offset by use of a filter bank with a large number  $N \gg 1$  of uniformly spaced channels each ( $j = 1..N$ ) of bandwidth  $B$ . The different channels can be averaged to give the cross-correlation as  $\sum_j \langle \overline{E}_j(t - \tau_f) \overline{E}_{j+k}^*(t + \delta t - \tau_{f+\delta f}) \rangle$  where  $j$  corresponds to  $f$  and  $k$  to  $\delta f = \text{const}$ . Techniques and software for extracting weak dispersed pulses from radio measurements have been highly developed for applications to pulsar searches (Ransom, Eikenberry, & Middleditch 2002; Ransom 2012).

A test and calibration of the proposed method can be obtained by applying it to the strong synchrotron emission of the Crab Nebula because in this case the dispersion measure is known from measurements of the Crab pulsar. It is DM

$\approx 56.77 \text{ pc(cm)}^{-3}$  (Lundgren et al. 1995). Inequality (7) then gives  $B < 1.46 f_{10}^{3/2} \text{ MHz}$ . The time delay in the cross-correlation between two frequencies separated by say  $\delta f = 5B$  is  $\delta t = -3.44 \mu\text{s}$ .

## 2.2 Interstellar/Intergalactic Scintillations

The interstellar scintillations (ISS) of small angular diameter extragalactic sources has been discussed by Lovelace & Backer (1972) and Goodman (1997). Lovelace and Backer considered the random modulation in the frequency domain of a wave from a broadband point source caused by the propagation through electron density irregularities in the interstellar medium (ISM). There is in general a “frequency correlation scale,”  $\Delta f$ , determined by the properties of the ISM. Two frequencies components of a wave from a point source separated by more than  $\Delta f$  are uncorrelated due to the random multi-path propagation. That is, the phase shift due to the extra propagation path length  $\pi f \Theta_s^2 Z/c$  differs by more than  $2\pi$  for frequencies separated by more than  $\Delta f$ , where  $\Theta_s$  is the scattering angle (proportional to  $f^{-2}$ ) and  $Z$  is the effective distance to the scattering screen. The value of  $\Delta f$  depends on the radio frequency and the propagation distance through the ISM. At frequencies  $f \lesssim f_* \approx 2 \text{ GHz}$ , the scintillations are strong and  $\Delta f/f \sim (f/f_*)^3$ , whereas at higher frequencies the scintillations are weak and  $\Delta f/f \sim 1$  (Lovelace & Backer 1972). Lovelace and Backer also estimated the critical angular size  $\psi$  of extragalactic radio sources to be of the order of  $25 \times 10^{-6} \text{ arc sec}$ . Sources of smaller size may exhibit random intensity variations whereas larger sources are steady. The calculated shift in the cross correlation function arises from the contribution of the individual radiating particles in the source and is not affected by the source size being larger or smaller than  $\psi$ .

Scintillations due to propagation through the inhomogeneous intergalactic medium are considered by Goodman (1997) and estimated to be small compared with the ISS.

In order for the shift in the cross-correlation function (equation 8) to be measurable we must have  $\delta f < \Delta f$ . This can be easily satisfied at GHz frequencies.

## 3 CONCLUSIONS

The detection of highly dispersed radio pulses likely of extragalactic origin by Lorimer et al. (2007) and Keane et al. (2012) opens the intriguing possibility that the sources of such pulses can be identified and thus their distances determined. This would allow direct measurements of the distributed baryon density of the intergalactic medium along lines-of-sight to possibly high redshift sources. This is of evident cosmological interest. Unfortunately, the narrow, highly dispersed radio pulses appear to be rare and non-repeating. Hence the distances to the sources are remain unknown.

More recently, a search by Bannister et al. (2012) for prompt radio pulses at 1.4GHz from gamma ray burst sources - some at cosmological distances - has revealed highly dispersed pulses. However, the association of the pulses with the gamma ray burst sources remains uncertain.

This work proposes a method to measure the dispersion measures of high-redshift incoherent synchrotron radio

sources associated with quasars by analysis of the amplitude cross-correlation of the radio waves received at two nearby frequencies. This method would allow direct measurements of the distributed baryon density of the intergalactic medium along lines-of-sight to sources of known high redshifts. This method utilizes the broadband nature of the synchrotron radiation. The bandwidths needed by this method are much narrower than those used to detect the mentioned dispersed radio pulses. Two key requirements on this method are identified: one is that the local oscillators at the two frequencies have very high stability (inequality 6) and the second is that the bandwidths be sufficiently narrow (inequality 10).

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